

Gauge invariant metric perturbations during reheating

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Abstract

The possible amplification of gauge invariant metric fluctuations in the infrared sector are very important during reheating stage of inflation. In this stage the inflaton field oscillates around the minimum of the scalar potential. The evolution for the super Hubble scales gauge invariant metric fluctuations can be studied by means of the Bardeen parameter. For a massive scalar field with potential $V(\Phi_c) = \frac{m^2}{2}\Phi_c^2 + \Lambda$ with nonzero cosmological constant. I find that (in the reheating regime and for super Hubble scales) the Sasaki-Mukhanov parameter oscillates with amplitude constant such that there is not amplification of Q during reheating.

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Postulating a period of nearly exponential growth in the primordial universe, inflationary cosmology [1] solved many problems which plagued previous models of the big bang. After inflation, the universe is dominated by the inflation, the scalar field whose evolution controls the dynamics of the inflationary era. The process for which the inflaton energy density is then converted into thermalized matter, is knowed as reheating [2]. The issue of gauge invariance becomes critical when we attempt to analyze how the scalar metric perturbations produced in the very early universe influence of a globally flat isotropic and homogeneous universe [3]. The evolution of gauge invariant metric perturbations during inflation have been well studied [4]. This allows to formulate the problem of the amplitude for the scalar metric perturbations on the evolution of the background Friedmann-Robertson-Walker (FRW) universe in a coordinate - independent manner at every moment in time. Parametric resonance instability occurs during a reheating period when the inflaton oscillates around the minimum of the scalar potential [5–7]. Since the gravitational perturbation

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is coupled to the inflaton field by the Einstein equation, it may also experience parametric amplification during this stage. The parametric resonance instabilities have important consequences for cosmology. They will lead to a reheating temperature which can be much larger than would be obtained by calculating the efficiency of reheating using perturbation theory and could have important implications for grand-unification-scale baryogenesis [8], the production of supermassive dark matter [9] or the formation of topological defects [10].

In this work I study the metric inhomogeneities during reheating for gauge invariant metric perturbations [3] by means of the Sasaki-Mukhanov variable [11] on super Hubble scales for a massive scalar field with potential $V(\Phi_c) = \frac{m^2}{2}\Phi_c^2 + \Lambda$ and nonzero cosmological constant and $\Phi_c = +\frac{3M_p H_0}{\pi m}$. Since the results do not depend on the gauge, the perturbed globally flat isotropic and homogeneous universe is well described by [3]

$$ds^2 = (1 + 2\psi) dt^2 - a^2(t)(1 - 2\Phi) dx^2, \quad (1)$$

where a is the scale factor of the universe and ψ and Φ the perturbations of the metric. I will consider the particular case where the tensor T_{ij} is diagonal, i.e., for $\Phi = \psi$ [12]. Linearizing the Einstein equations in terms of the matter and metric fluctuations ϕ and Φ , one obtains the system of differential equations for ϕ and Φ [13]

$$\ddot{\Phi} + \left(\frac{\dot{a}}{a} - 2\frac{\ddot{\phi}_c}{\dot{\phi}_c} \right) \dot{\Phi} - \frac{1}{a^2} \nabla^2 \Phi + 2 \left[\frac{\ddot{a}}{a} - \left(\frac{\dot{a}}{a} \right)^2 - \frac{\dot{a}}{a} \frac{\ddot{\phi}_c}{\dot{\phi}_c} \right] \Phi = 0, \quad (2)$$

$$\frac{1}{a} \frac{d}{dt} (a\Phi)_{,\beta} = \frac{4\pi}{M_p^2} (\dot{\phi}_c \phi)_{,\beta}, \quad (3)$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{1}{a^2} \nabla^2 \phi + V''(\phi_c)\phi + 2V'(\phi_c)\Phi - 4\dot{\phi}_c\dot{\Phi} = 0. \quad (4)$$

Here, the dynamics of the spatially homogeneous field ϕ_c being described by the equations [14]

$$\ddot{\phi}_c + 3\frac{\dot{a}}{a}\dot{\phi}_c + V'(\phi_c) = 0, \quad (5)$$

$$\dot{\phi}_c = -\frac{M_p^2}{4\pi} H'_c(\phi_c), \quad (6)$$

where the prime denotes the derivative with respect to ϕ_c and $H_c(\phi_c) \equiv \frac{\dot{a}}{a}$.

In the case of adiabatic perturbations with scales outside the Hubble radius, it is convenient to work with the Bardeen parameter

$$\zeta = \frac{2}{3} \frac{\Phi + H_c^{-1} \dot{\Phi}}{1 + \omega} + \Phi, \quad (7)$$

where the overdot denotes the time derivative and $\omega = p/\rho$ is the ratio of the pressure to the density of the background. In the $k \rightarrow 0$ limit, the Bardeen parameter satisfies [16,12]

$$\frac{3}{2}(1 + \omega)H_c\dot{\zeta} = 0, \quad (8)$$

which, in the reheating stage holds the following condition [12]

$$\dot{\phi}_c^2 \dot{\zeta} = 0. \quad (9)$$

During the slow roll regime the eq. (2) is well behaved, but during the reheating period the field ϕ_c oscillates and the coefficients in eq. (2) has singularities, which can be removed by means of the Sasaki-Mukhanov variable $Q = \phi + \frac{\dot{\phi}_c}{\dot{H}_c} \Phi$. Hence, from eq. (2), the modes for Q satisfies [17]

$$\ddot{Q}_k + 3H_c \dot{Q}_k + \left[V'' + \frac{k^2}{a^2} + 2 \frac{d}{dt} \left(\frac{\dot{H}_c}{H_c} + 3H_c \right) \right] Q_k = 0. \quad (10)$$

The parameter Q is related with the Bardeen one by the following relation [18]

$$\zeta = \frac{H_c}{\dot{\phi}_c} Q. \quad (11)$$

The structure of the eq. (10) can be simplified with the map $Q = e^{-3/2 \int H_c dt} P$, such that the equation for the modes of P is

$$\ddot{P}_k + \left[\frac{k^2}{a^2} + V'' + \frac{9}{2} \dot{H}_c - \frac{9}{4} H_c^2 + 2 \frac{\ddot{H}_c}{H_c} - 2 \left(\frac{\dot{H}_c}{H_c} \right)^2 \right] P_k = 0, \quad (12)$$

which can be written as $\ddot{P}_k + \left[\frac{k^2}{a^2} - \frac{k_0^2}{a^2} \right] P_k = 0$, where $\mu^2(t) = \frac{k_0^2}{a^2}$ is an effective parameter of mass given by

$$\mu^2(t) = 2 \left(\frac{\dot{H}_c}{H_c} \right)^2 - V'' - \frac{9}{2} \dot{H}_c - \frac{9}{4} H_c^2 - 2 \frac{\ddot{H}_c}{H_c}. \quad (13)$$

In the infrared sector one takes $\frac{k^2}{a^2} \ll \frac{k_0^2}{a^2}$, where k_0 is the time dependent wavenumber which separates the infrared and ultraviolet sectors. During inflation, for a given scalar potential $V(\phi_c)$, the dynamics of the Hubble parameter $H_c(\phi_c)$ being described by the Friedmann equation (see, for example, [15])

$$V(\phi_c) = \frac{3M_p^2}{8\pi} \left[H_c^2 - \frac{M_p^2}{12\pi} \left(\frac{\dot{H}_c}{\dot{\phi}_c} \right)^2 \right]. \quad (14)$$

In the reheating period the second term inside the brackets cannot be neglected because the slow roll conditions are violated.

Now I consider the scalar potential $V(\Phi_c) = \frac{m^2}{2} \Phi_c^2 + \Lambda$, where Λ is some negative cosmological constant and $\Phi_c = \phi_c + \frac{3M_p H_0}{\pi m}$. If $\Phi_c(t) = \frac{2}{5} M_p \cos(\omega\tau)$ [here $\tau = t - t(\phi_c = 0)$], one obtains $H_c = 4\sqrt{\frac{\pi}{75}} m \cos(\omega\tau) + H_0$, $V'' = m^2$ and $\dot{H}_c = -4\sqrt{\frac{\pi}{75}} m \omega \sin(\omega\tau)$. Here, ω is the oscillation frequency of the inflaton field and $H_0 \equiv H_c(\phi_c = 0)$. From eq. (14) one obtains the cosmological constant

$$\Lambda = -\frac{M_p^2}{2\pi} \left[H_0^2 \left(\frac{36 - 3\pi}{4\pi} \right) + \frac{m^2}{12\pi} \right]. \quad (15)$$

This case was studied by Finelli and Branderberger [18] but without cosmological constant. In this paper I am interested in the study of possible consequences in the evolution of super Hubble metric perturbations during reheating when one take into account a nonzero cosmological constant. During reheating $0 < H_c < m$, so that $m > H_0 > \frac{4}{5}m$ and Λ can take negative values near the minimum of the potential. The effective parameter of mass is

$$\mu^2(\tau) = 2 \left(\frac{4\sqrt{\frac{\pi}{75}}m\omega\sin(\omega\tau)}{4\sqrt{\frac{\pi}{75}}m\cos(\omega\tau) + H_0} \right)^2 + 18\sqrt{\frac{\pi}{75}}[\sin(\omega\tau) - \cos(\omega\tau)] - \frac{12\pi}{25}m^2\cos^2(\omega\tau) - \frac{9}{4}H_0^2 + \frac{8\sqrt{\frac{\pi}{75}}m\omega^2\cos(\omega\tau)}{4\sqrt{\frac{\pi}{75}}m\cos(\omega\tau) + H_0}. \quad (16)$$

Since in the infrared sector $\frac{k^2}{a^2} \ll \frac{k_g^2}{a^2}$, the solution for P_k is given with a very good approximation for super Hubble scales by the equation $\ddot{P}_k - \mu^2(\tau)P_k = 0$. For inflation take place one requires that $\mu^2(\tau) \geq 0$. The conditions required are (for $\omega < 1$; $m = 10^{-5}M_p$)

$$\frac{\omega}{m} > 1, \quad (17)$$

$$H_0 - 4\sqrt{\frac{\pi}{75}}m \ll 10^{-1}M_p. \quad (18)$$

Figure 1 shows the numerical solution for the evolution of $P e^{-\frac{3}{2}H_0t} \equiv Q e^{\frac{3m}{2\omega}\sin(\omega\tau)}$ for $\tau \geq 0$ and $\omega = 10^{-3}M_p$. Note that the amplitude of $Q e^{\frac{3m}{2\omega}\sin(\omega\tau)}$ increases with time. This means that Q also increases with time. Note that in this case $\mu^2 < 0$ and inflation do not take place.

Figure 2 shows $P e^{-\frac{3}{2}H_0t}$ for $\omega = 10^{-5}M_p = m$. Note that $Q e^{\frac{3m}{2\omega}\sin(\omega\tau)}$ oscillates with amplitude constant and effective frequency $\omega_{eff} \simeq 6 \times 10^{-3}M_p$. Hence, the solution for the Sasaki-Mukhanov variable will be a coupled oscillator with frequencies $\omega = 10^{-5}M_p$ and $\omega_{eff} \simeq 6 \times 10^{-3}M_p$. In this case $\mu^2 > 0$ and $\mu^2 \gg \frac{k^2}{a^2}$ in the IR sector.

Finally, from eq. (11) one obtains the Bardeen parameter for super Hubble scales during the reheating era

$$\zeta = -\frac{5}{2} \left[\frac{4\sqrt{\frac{\pi}{75}}m\cos(\omega\tau) + H_0}{M_p\omega\sin(\omega\tau)} \right] Q. \quad (19)$$

Note that the denominator in eq. (19) has singular points in $\omega\tau = n\pi$ ($n = 0, 1, 2, \dots$). Hence, the Bardeen parameter indeterminates for $\omega\tau = n\pi$. These singularities in ζ correspond to the the points where $\dot{\phi}_c = 0$. This result disagrees with other results of a previous work [19], where the authors founded that ζ oscillates with constant amplitude during reheating.

The results here obtained for Q are very similar with the Finelli and Branderberger [18] (i.e., for $\mu^2 > 0$) because there is not growth of the Sasaki-Mukhanov variable for super Hubble scales during reheating. The only difference is that here Q oscillates around a constant value. This oscillation should be a consequence that — instead Finelli and Branderberger have made to simplify the equation of motion for P_k (12) — I worked without perform a time average in the effective parameter of mass [see eq. (16)]. The calculation without a time average leads to an oscillating μ^2 which is the responsible for the oscillation of Q . A similar result, but when super Hubble non-adiabatic pressure perturbation is negligible was obtained recently by Wands *et. al.* [20].

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FIGURES

FIG. 1. Evolution for $P/a^{3/2}$ as a function of τ , for $H_0 = 0.4m$, $m = 10^{-5} M_p$ and $\omega = 10^{-3} M_p$. The amplitude of $P/a^{3/2}$ increases with time in the ultraviolet sector. Note that we have used Planckian unities.

FIG. 2. Evolution for $P/a^{3/2}$ as a function of τ , for $H_0 = 0.4m$, $m = 10^{-5} M_p$ and $\omega = 10^{-5} M_p$. The amplitude of $P/a^{3/2}$ remains constant. Note that $P/a^{3/2}$ oscillates with effective frequency $\omega_{eff} \simeq 6 \times 10^{-3} M_p$.